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Regional water balance modelling in the NOPEX area: development and application of monthly water balance models

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Abstract

One of the main purposes of a water balance study is to evaluate the net available water resources, both on the surface and in the subsurface. Water balance models that simulate hydrographs of river flow on the basis of available meteorological data would be a valuable tool in the hands of the planners and designers of water resources systems. In this paper, a set of simple monthly snow and water balance models has been developed and applied to regional water balance studies in the NOPEX area. The models require as input monthly areal precipitation, monthly long-term average potential evapotranspiration and monthly mean air temperature. The model outputs are monthly river flow and other water balance components, such as actual evapotranspiration, slow and fast components of river flow, snow accumulation and melting. The results suggest that the proposed model structure is suitable for water balance study purposes in seasonally snow-covered catchments located in the region.

1. Introduction

Water balance investigations on different time and spatial scales are one of the major objectives of the NOPEX (A NOrthern hemisphere climate Processes land-surface EXperiment) project (Lundin and Halldin, 1994). Quantitative estimates usually require modelling, since simulation is one of the most widely used techniques in operations research and management science (Bouraoui and Wolfe, 1990). Water balance models were first developed in the 1940s by Thornthwaite (1948) and later revised by Thornthwaite and Mather (1955, 1957). Since then, water balance techniques have been adopted, modified, and applied to a diversity of hydrological

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problems (Alley, 1984; Xu, 1992; Vandewiele et al., 1992; Xu and Vandewiele, 1994, 1995); they have proved to be both flexible and understandable and have been computed on daily, weekly, monthly, and even annual bases (Alley, 1984). Under many conditions, they provide accurate estimates of surface runoff when compared with measured streamflow, accurate measures of relative changes in soil moisture, reliable evapotranspiration estimates, and estimates of ground water discharge and recharge rates (Gleick, 1987).

This paper describes the development and application of simple regional water balance models with a monthly time step. This is the first stage in a modelling strategy within NOPEX where two ultimate goals are to relate physiographical information to catchment dynamics without the use of measured runoff time series and to properly account for subgrid-scale processes in a regional context. We consider it logical to approach these goals in discrete stages. At the present level of knowledge of hydrological processes modelling is first to identify the dominant processes affecting the output that require most accuracy and to improve the level of simulation of the chosen processes, i.e. following the principle of the progressive modification (Nash and Sutcliffe, 1970), progressing from the simple towards the more complex, only when a sufficiently better fit justifies the additional complexity. The simplicity here is not a virtue in itself but is a pragmatic response to a desire to produce a modelling approach that is capable of being applied operationally, whilst reflecting the necessary accuracy and physical relevance. The input data required are monthly areal precipitation, the monthly long-term average potential evapotranspiration and monthly mean air temperature. The model outputs are monthly river flow and other water balance components, such as actual evapotranspiration, slow and fast components of river flow, soil moisture storage and accumulation of snowpack, etc. Such information is

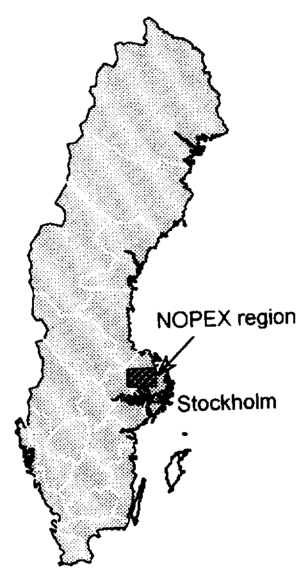


Fig. 1. Map of Sweden with the location of the NOPEX region.

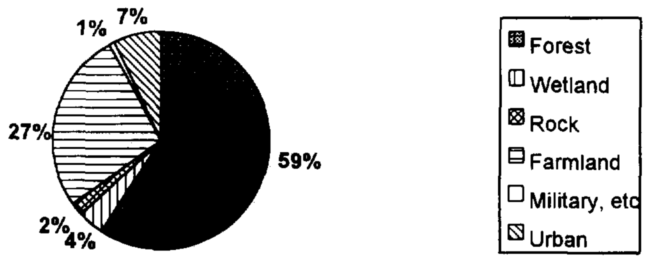


Fig. 2. Land use within the NOPEX region (reproduced from Lundin and Halldin, 1994).

likely to have important ramifications for long-range water resources planning. As an additional benefit, the resulting estimates of internal series, like evapotranspiration and soil moisture storage, could possibly be used for calibrating satellite imagery.

2. Study region and data

2.1. Location of the NOPEX research area

The NOPEX region, an area about 60 km × 90 km north of Uppsala in the

Table 1
Precipitation stations (from Seibert, 1994)

Name	Latitude	Longitude	Level (m.a.s.l.)	Correction (%)	Station number
Drälinge	59°59′	17°34′	30	23	9759
Enköping	59°38′	17°04′	20	16	9738
Films Kyrkby	60°14′	17°54′	39	18	10714
Kolkärna[a]	60°10′	16°19′	68	19	10610
Frötuna	59°54′	17°52′	15	18	9755
Gysinge	60°17′	16°53′	63	20	10617
Hallstaberg	59°39′	16°47′	25	22	9639
Harbo	60°08′	17°14′	40	20	10708
Hyvlinge	59°44′	17°04′	20	18	9745
Österby	59°40′	17°41′	10	22	9740
Sala[b]	59°54′	16°40′	60	18	9655
Skultuna	59°43′	16°26′	40	21	9644
Sundby	59°41′	16°40′	35	22	9641
Tärnsjö	60°08′	16°56′	55	20	10612
Ultuna	59°49′	17°38′	15	27	9749
Uppsala Fly.[a]	59°53′	17°35′	21	24	9753
Västerås-H.[a]	59°35′	16°37′	06	27	9635
Vattholma	60°01′	17°44′	25	18	10701
Vittinge	59°54′	17°01′	50	21	9754

[a] Stations with temperature observations and monthly long-term average potential evapotranspiration.
[b] Station with temperature observations.

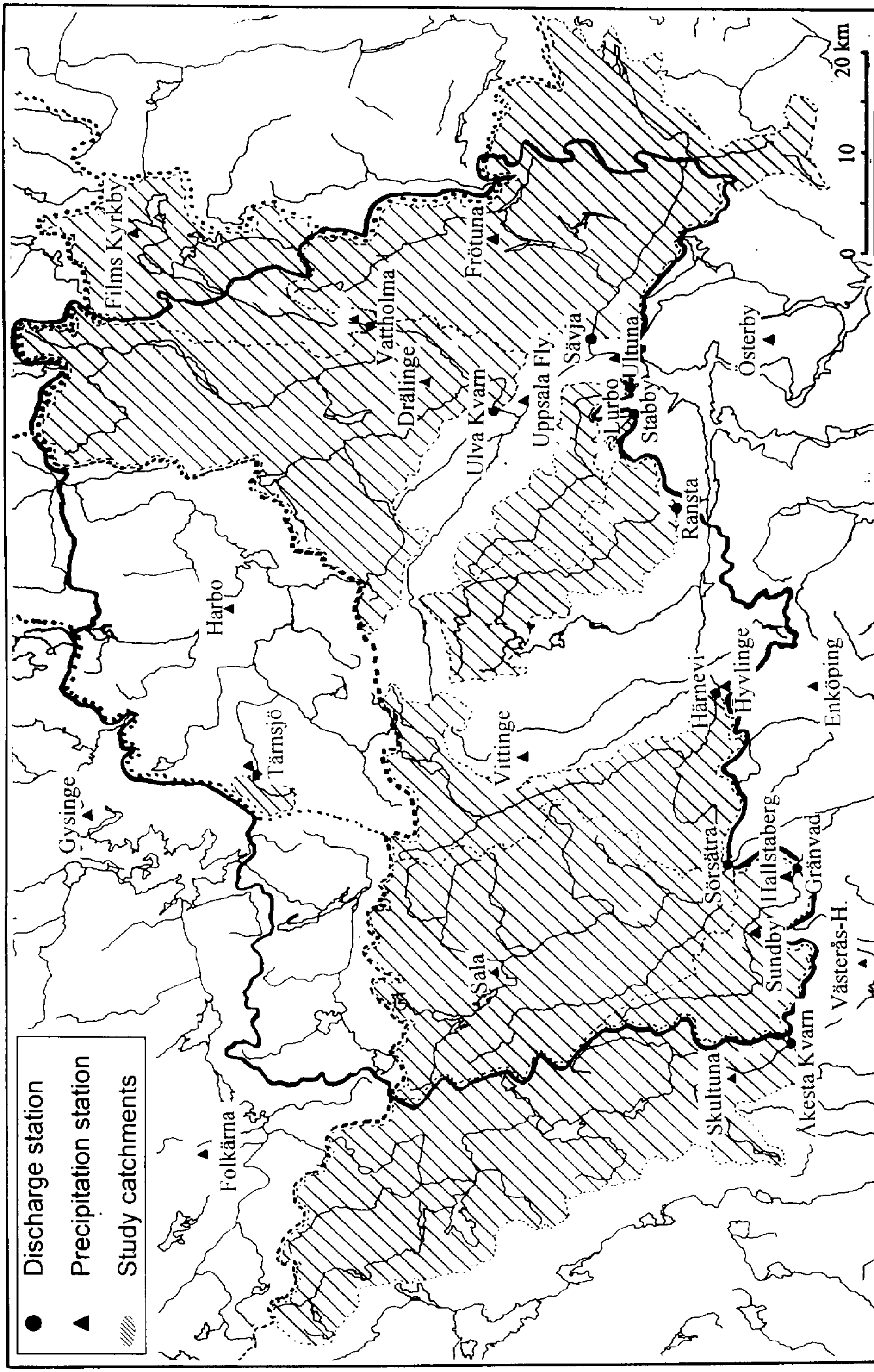


Fig. 3. Locations of the catchments and precipitation and discharge stations. The NOPEX area is encircled by thick solid line (reproduced from Seibert, 1994).

southern part of the boreal forest zone, is characterised by an irregular mixture of forest and agriculture (Fig. 1).

The landscape in the NOPEX area is dominated topographically and morphologically by a very flat subcambrian peneplane. The geology is characterised by granite, sedimentary gneiss and leptite. Clayey soils and moraines dominate in the area. The fine-grained clay soils together with areas of sandy and silty material dominate in the south. Areas with moraine and bogs predominate more and more towards the northern part of the area. The region is crossed by five north–southerly oriented eskers. Height differences are small, with the main part of the region confined between 30 and 70 m above sea-level. Extreme values are 1 and 131 m above sea-level.

Forest dominates the area as a whole (Fig. 2), agricultural fields are concentrated towards the south and a continuously greater fraction of the ground is covered with forests towards the north.

Table 2
The basin characteristics (from Seibert, 1994)

Station	Abbreviation	River	Coordinates	Station number
Gränvad	Gr	Lillån	661637 155504	61-2217
Härnevi	Ha	Örsundaån	662438 157112	61-2248
Lurbo	Lu	Hågaån	663271 160107	61-2245
Ransta	Ra	Sävaån	662754 158926	61-2247
Sävja	Sa	Sävjaån	663592 160652	61-2243
Sörsätra	So	Sagån	662278 155498	61-2220
Stabby	St	Stabbybäcken	663200 159982	61-1742
Tärnsjö	Ta	Stalbobäcken	666859 156333	54-2299
Ulva Kvarn	Ul	Fyrisån	664509 159902	61-2246
Vattholma	Va	Vattholmaån	665713 160736	61-2244
Åkesta Kvarn	Ak	Svartån	661722 153742	61-2216

Station	Area (km^2)	Lake (%)	Forest (%)	Field or meadow (%)	Altitude min (m.a.s.l.)	Altitude max (m.a.s.l.)
Gränvad	168.0	0	41.0	59.0	15	75
Härnevi	305.0	1.0	55.0	44.0	15	105
Lurbo	124.0	0.3	77.7	27.0	15	75
Ransta	198.0	0.9	66.1	33.0	15	105
Sävja	727.0	2.0	64.0	34.0	5	75
Sörsätra	612.0	1.1	61.0	37.9	35	145
Stabby	6.6	0	87.0	13.0	18	55
Tärnsjö	14.0	1.5	84.5	14.0	55	105
Ulva Kvarn	950.0	3.0	61.0	36.0	5	95
Vattholma	284.0	4.8	71.0	24.2	25	65
Åkesta Kvarn	730.0	4.0	69.0	27.0	25	215

2.2. Data used in the study

In this study, 11 gauged catchments in the NOPEX area ranging in size from 6.6 to 950 km^2 were used. There are 19 precipitation stations inside and/or close to the region, four of which also measure daily air temperature. Monthly long-term average potential evapotranspiration (1961–1978) data calculated from the Penman equation are available at three stations within the area (Eriksson, 1981). These stations are listed in Table 1, and Fig. 3 shows the locations of the catchments. The basin characteristics are shown in Table 2.

Eleven years (1981–1991) of monthly data were used in this study. The computation of the discharge at time $t = 1$ needs a knowledge of the initial storage at $t = 0$, the calibration period has to be preceded by a 'warm-up' period where the input series are known and which enables the computation of the initial storage starting with an arbitrary storage. The first year (1981) was used as a 'warm-up' period.

According to Eriksson (1983) the Swedish Meteorological and Hydrological Institute (SMHI) gauges underestimate, systematically, the yearly precipitation by 20–25%, because of losses owing to the wind field around the gauge, adhesion at the instrument and evaporation from the gauge. Eriksson (1983) and Seibert (1994) calculated correction factors for the precipitation stations (Table 1).

Areal precipitation for each catchment has been calculated by Seibert (1994) using

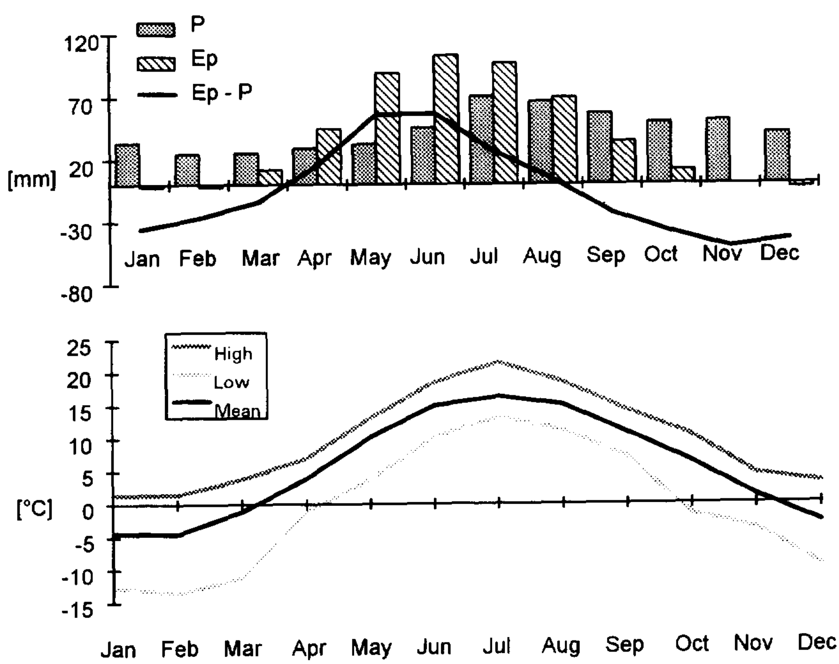


Fig. 4. The seasonal variations of the major meteorological variables (1960–1989) for Uppsala. Top diagram shows precipitation (P) and potential evapotranspiration (Ep) as well as the difference between them. Bottom diagram shows air temperature, together with the historical (1739–1989) maxima and minima (reproduced from Lundin and Halldin, 1994).

the Thiessen method. In view of the geographical locations of the temperature stations, temperature data at station Uppsala Fly. were used for catchments Ra, St, Lu, Ul, Sa and Va, while the arithmetic mean of the other three temperature data series was used for catchments Ak, Gr, Ha, So and Ta. Likewise, monthly long-term average potential evapotranspiration data at station Uppsala Fly. were used recursively to form an 11 year sequence concurrent with the precipitation and temperature for catchments Ra, St, Lu, Ul, Sa and Va, while the arithmetic mean of data from the two other evaporation stations was used for catchments Ak, Gr, Ha, So and Ta. All quantities are expressed in millimetres per month, except temperature, which is in degrees Celsius. The seasonal variations of the mean monthly values of the major meteorological variables are shown in Fig. 4.

3. Model concept

In view of the aims of the research project set out earlier, the requirements of the developed models should be simple in structure and have as few parameters as possible, so that the relationship between model parameters and physical features can be established with a certain level of confidence; "keeping the number of parameters as low as possible increases the information content per parameter and therefore allows both a more accurate determination of the parameters and a more

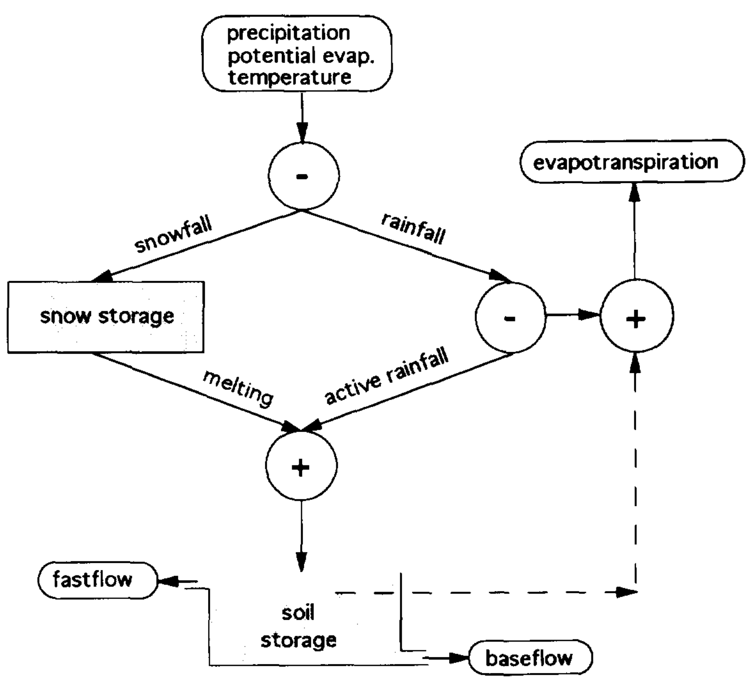


Fig. 5. The concept of the monthly snow and water balance model.

reliable correlation of the values obtained with catchment characteristics" (Dooge, 1977).

The concept of the model is shown in Fig. 5. Precipitation p_t is first split into rainfall r_t and snowfall s_t by using a temperature index function similar to that described by Vandewiele and Ni-Lar-Win (1993). Snowfall is added to the snowpack sp_t (the first storage) at the end of the month, of which a fraction m_t melts and contributes to the soil moisture storage sm_t. Before the rainfall contributes to the soil storage as 'active' rainfall, a small part is subtracted and added to evapotranspiration loss. The latter storage contributes to evapotranspiration e_t, to the fast component of flow f_t following the variable source area concept and to base flow b_t.

4. Model equations

4.1. Snow accumulation and melting

Whether precipitation falls as rain or snow has a significant influence on the formation of runoff and other water balance terms. Model performance is therefore sensitive to decisions made concerning the form of precipitation and snowmelt. In the literature, two approaches were used concerning snow separation and melt. The first approach uses the energy balance method, as discussed by, for example, US Corps of Engineers (1956, 1960), Forsman (1963), Kuz'min (1972) and Gray and Landine (1988). The variables necessary for a complete heat budget computation are total solar radiation, albedo, longwave radiation balance (effective radiation), air temperature, air humidity, wind speed, temperature gradients in the soil and snow and precipitation. In addition, some physical parameters governing heat exchange with the atmosphere, heat transfer within the snowpack, liquid water content in the snow and drainage of snowpack, have to be estimated.

In view of the data that are generally available in a catchment, detailed heat budget computations are hardly warranted. The great inhomogeneities in the areal distribution of snow cover complicates the picture further. Rather crude index methods, i.e. various forms of degree-day methods based on a threshold temperature, are therefore often preferred in operational models (Bengtsson, 1976; Granger and Male, 1978; Linsley, 1982).

When developing the snowroutine for the water balance model, recordings of air

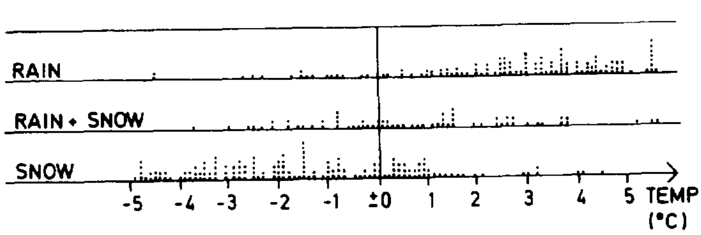


Fig. 6. The observer's note on the form of precipitation related to mean daily air temperature. Each point represents 1 day with precipitation (at Lilla Tivsjön Station, Sweden; Bergström, 1976).

temperature and precipitation have been the main input variables and it was felt that the form of precipitation as observed in the field (Fig. 6) could be reasonably represented by the following functions.

This new method works as follows: the solid part (snow) s_t of precipitation p_t is a fraction of p_t and depends on temperature c_t, as

$$s_t = p_t\{1 - \exp[-(c_t - a_1)/(a_1 - a_2)]^2\}^+ \quad (1)$$

where a_1 and a_2 are two unknown constants to be estimated, so-called parameters with constraint $a_1 > a_2$. The plus sign means $x^+ = \max(x, 0)$. This relationship is represented in Fig. 7.

The snowpack balance is written as

$$sp_t = sp_{t-1} + s_t - m_t \quad (2)$$

where m_t, the snowmelt during month t, is considered as a function of temperature c_t and snowpack sp_{t-1} at the beginning of month t, as

$$m_t = sp_{t-1}\{1 - \exp[(c_t - a_2)/(a_1 - a_2)]^2\}^+ \quad (3)$$

Fig. 7 also illustrates this relationship. Snowfall and melting can occur during the same month owing to the lumping of time and space. Subdivision of precipitation leads to

$$r_t = p_t - s_t \quad (4)$$

where r_t is the rainfall.

4.2. Computation of evapotranspiration

For computing actual evapotranspiration e_t, two factors, among others, were considered to be most important: the monthly potential evapotranspiration ep_t,

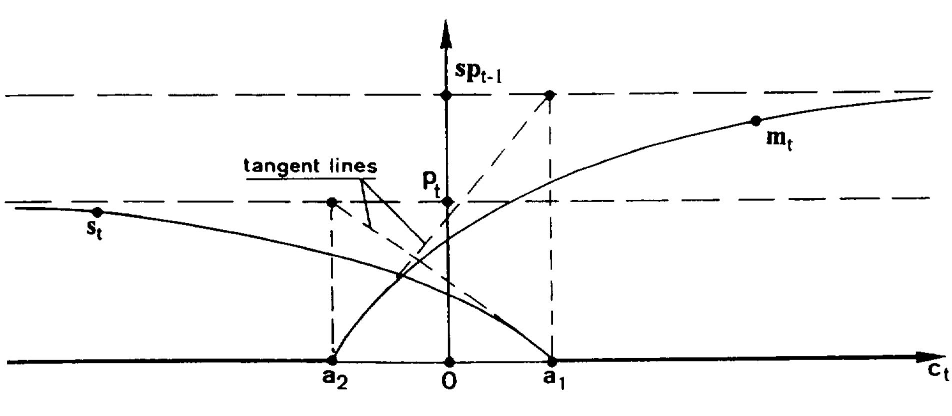


Fig. 7. Snowfall s_t and melting m_t during month t as a function of temperature c_t.

and the available water w_t during month t, defined as

$$w_t = r_t + sm_{t-1}^+ \tag{5}$$

where $sm_{t-1}^+ = \max\ (sm_{t-1}, 0)$ is the available storage.

In order to improve the model performance when the spring and summer was much colder or warmer than normal, the monthly long-term average potential evapotranspiration was corrected on the basis of mean monthly temperatures and long-term averages according to Eq. (6)

$$ep_t = [1 + a_3(c_t - c_{\mathrm{m}})]ep_{\mathrm{m}} \tag{6}$$

where: ep_t is the adjusted monthly potential evapotranspiration, a_3 is a model parameter, c_t is the monthly mean air temperature, c_{m} is the monthly long-term average temperature, and ep_m is the monthly long-term average potential evapotranspiration.

For evident reasons, a good evapotranspiration equation must be such that

$$\left.\begin{array}{l} e_t \text{ increases with } ep_t \text{ and } w_t \\ e_t = 0 \quad \text{when } w_t = 0 \text{ or } ep_t = 0 \\ e_t \leqslant ep_t \quad \text{and } e_t \leqslant w_t \\ e_t \to ep_t \text{ when } w_t \to \infty \end{array}\right| \tag{7}$$

Two equations were used in the model. The first one is

$$e_t = \min\ [ep_t(1 - a_4^{w_t/ep_t}), w_t] \tag{8}$$

where a_4 is a parameter, characteristic of the river basin under study. It is constrained by $0 \leqslant a_4 < 1$ because of the condition (7).

The second equation is

$$e_t = \min\ \{w_t[1 - \exp(-a_4 ep_t)], ep_t\} \tag{9}$$

with parameter a_4 constrained by $a_4 \geqslant 0$. Both Eqs. (8) and (9) fulfil condition (7).

4.3. Computation of slow flow

Being similar to baseflow, slow flow depends essentially on the storage in the catchment during the month considered. The general form of the slow flow equation is

$$b_t = a_5(sm_{t-1}^+)^{b_1} \tag{10}$$

where a_5 and b_1 are non-negative parameters. In practice, however, a_5 and b_1 are highly correlated resulting in difficulty in calibration and high imprecision of the estimates. Therefore b_1 was given one of three standard values, namely, $b_1 = 1/2$ or 1 or 2, of which at least one value suits any given river basin. Thus b_1 is a discrete parameter, while a_5 is a continuous one (free parameter).

Table 3
Quality and residual seasonality

Catchment	Models							
	1	2	3	4	5	6	7	8
AK	2.22-0	2.40-0	2.42-0	2.50-0	2.41-0	2.53-0	2.52-0	2.61-0
GR	2.24-2	2.37-0	2.33-2	2.43-1	2.44-1	2.46-1	2.49-1	2.52-1
HA	2.49-0	2.57-0	2.61-0	2.65-0	2.64-0	2.63-0	2.68-0	2.70-0
LU	2.02-0	2.09-0	2.14-0	2.14-0	2.10-0	2.13-0	2.15-0	2.19-0
RA	2.50-0	2.61-0	2.65-0	2.73-0	2.54-0	2.55-0	2.58-0	2.65-0
SA	2.28-0	2.49-0	2.48-1	2.66-0	2.39-0	2.42-0	2.40-1	2.53-1
SO	2.06-0	2.04-0	2.14-0	2.11-0	2.09-0	2.05-0	2.13-0	2.12-0
ST	1.86-0	1.84-0	1.88-1	1.86-1	1.97-0	1.94-0	1.99-1	1.96-0
TA	2.29-0	2.33-0	2.36-2	2.40-1	2.24-0	2.29-0	2.26-1	2.34-1
UL	2.24-1	2.47-1	2.45-2	2.64-1	2.37-0	2.57-1	2.55-1	2.71-1
VA	2.24-0	2.51-0	2.72-1	2.84-1	2.40-0	2-53-0	2.74-1	2.81-1

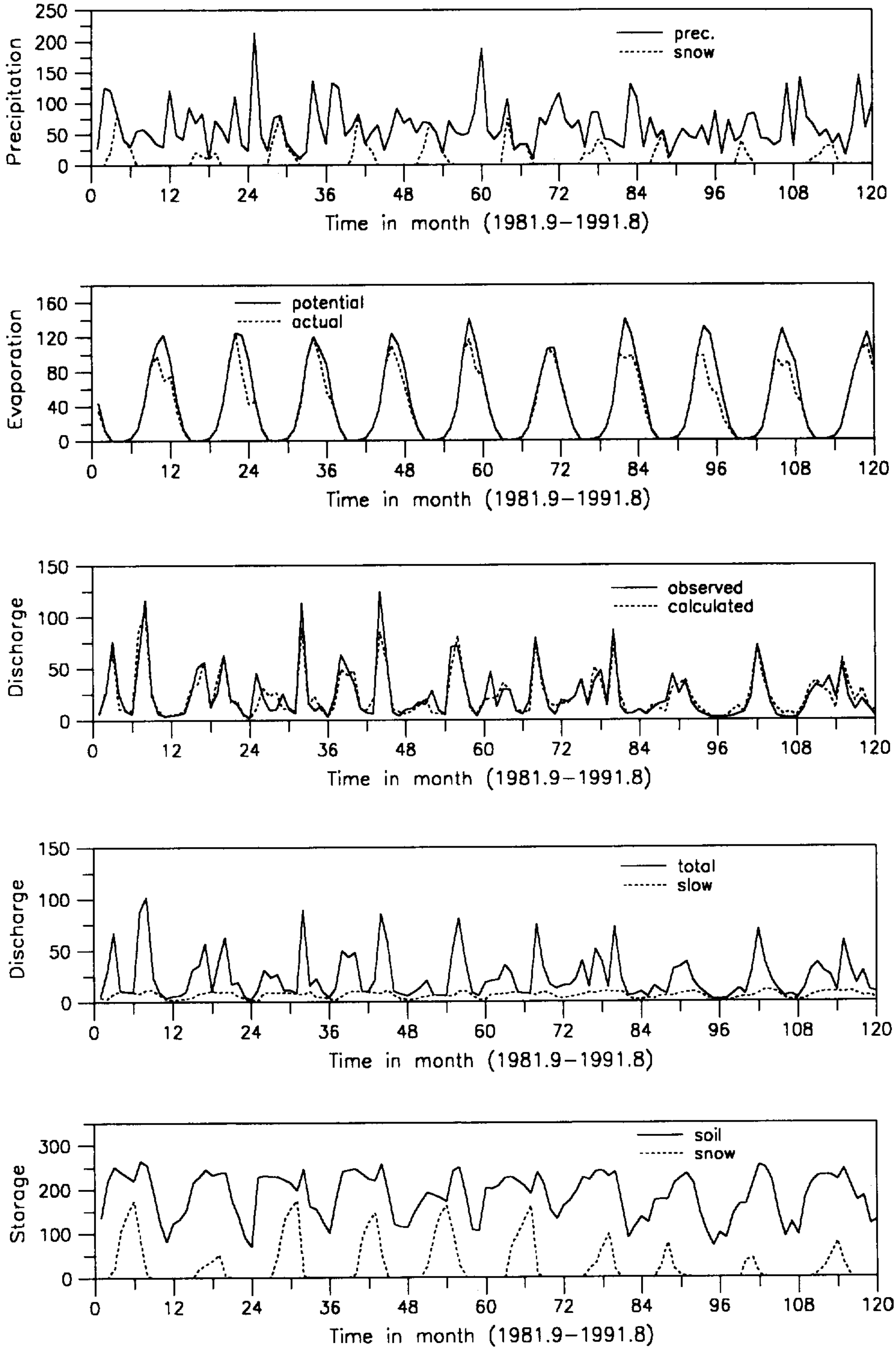
The first number is quality and the second number is seasonality.
Double underline means the best model, single underline means good models.

4.4. Computation of fast flow

Fast flow depends on rainfall, r_t, on the snowmelt, m_t, on other meteorological conditions as reflected by ep_t, on the state of the basin as measured by storage, sm_t and on the physical characteristics of the basin, which are taken into account by the introduction of parameters.

Table 4
Optimised parameter values of model 5 for the 11 catchments

Catchment	Parameter values					
	a_1	a_2	a_3	a_4	a_5	a_6
AK	3.20	−1.48	0.120	0.0111	0.0757	0.00327
GR	1.13	−2.38	0.059	0.0121	0.0375	0.00408
HA	1.61	−2.24	0.077	0.0100	0.0585	0.00420
LU	1.46	−2.15	0.097	0.0111	0.0854	0.00476
RA	1.72	−1.81	0.091	0.0108	0.0635	0.00400
SA	1.82	−2.06	0.085	0.0159	0.0690	0.00337
SO	1.49	−2.09	0.201	0.0092	0.0812	0.00564
ST	1.74	−2.40	0.085	0.0110	0.0254	0.00496
TA	2.79	−1.77	0.101	0.0063	0.0643	0.00180
UL	2.38	−1.72	0.030	0.0164	0.0610	0.00250
VA	2.61	−1.40	0.070	0.0153	0.1160	0.00266


Precipitation
prec.
snow
Time in month (1981.9–1991.8)
Evaporation
potential
actual
Time in month (1981.9–1991.8)
Discharge
observed
calculated
Time in month (1981.9–1991.8)
Discharge
total
slow
Time in month (1981.9–1991.8)
Storage
soil
snow
Time in month (1981.9–1991.8)

A useful quantity is the 'active' rainfall, defined as

$$n_t = r_t - ep_t[1 - \exp(-r_t/ep_t)] \tag{11}$$

The fast runoff equation takes the form

$$f_t = a_6(sm_{t-1}^{+})^{b_2}(m_t + n_t) \tag{12}$$

where a_6 and b_2 are non-negative parameters. As in the case of slow flow, b_2 was given one of three standard values, namely, $b_2 = 1/2$ or 1 or 2.

Equation (12) can be seen as a translation of the variable source area concept: the greater the storage, sm_{t-1}^{+}, the wetter the catchment, the greater the 'source' of fast runoff, the greater the part of the 'active' rainfall and snowmelt running off rapidly.

Since sm_t varies slowly in contrast with m_t and n_t, Eqs. (10) and (12) really represent slow and fast flow, respectively.

The total computed discharge is

$$d_t = b_t + f_t \tag{13}$$

The soil moisture storage at the end of the month t is updated by a water balance equation

$$sm_t = sm_{t-1} + r_t + m_t - e_t - d_t \tag{14}$$

By specifying values of b_1 and b_2 and by the choice of the evapotranspiration Eqs. (8) and (9), one finds what will be called henceforth a particular model. Since there are three possible values of b_1 and b_2, and two possible evapotranspiration equations, there are $2 \times 3 \times 3 = 18$ possible models defined by Eqs. (1)–(14), which reflect a large flexibility. A programme is available to find, automatically, the 'best' model for a given catchment. Because the models have very simple structure and deal with monthly data, the amount of computer work is very small. The whole procedure for optimising the 18 possible models needs only about 5 min of execution time on an IBM PC-486 computer.

5. Statistical analysis

Statistical analysis was confined to non-linear regression analysis. In the present section, only a limited number of items are discussed. A full account is given by Xu (1992) and Vandewiele et al. (1992, 1993).

Fig. 8. Precipitation, snowfall, potential and actual evapotranspiration, computed and observed discharges, total and slow discharge and soil moisture and snow storage vs. time diagrams (Ha, model 5). All quantities are expressed in millimetres per month.

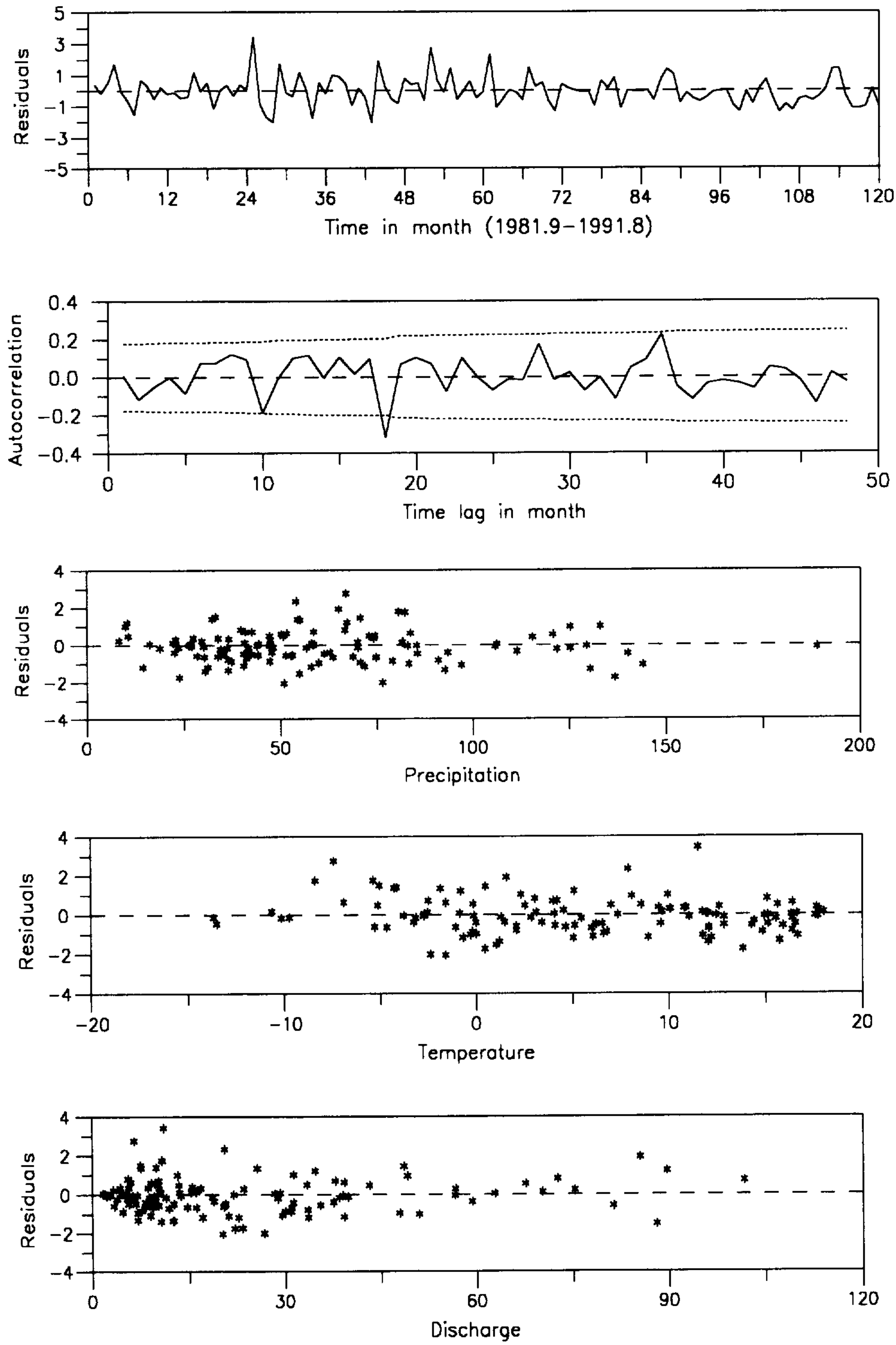

Residuals
Time in month (1981.9-1991.8)
Autocorrelation
Time lag in month
Residuals
Precipitation
Residuals
Temperature
Residuals
Discharge

Table 5
Summary of calibration results (model 5) for the HA catchment

Calibration results	
Evapotranspiration equation	(9)
Discrete slow flow parater b_1	1
Discrete fast flow parameter b_2	1
Model standard deviation σ	0.983
Half width of 95% confidence interval of σ	0.127
Mean computed flow (mm month^{-1})	23.88
Contribution of slow flow (%)	27
Contribution of fast flow (%)	73
Observed quantities	
Mean precipitation (mm month^{-1})	61.50
Mean runoff (mm month^{-1})	24.17
Runoff coefficient (%)	39.00

Table 6
Parameter estimates of the Ha catchment (model 5)

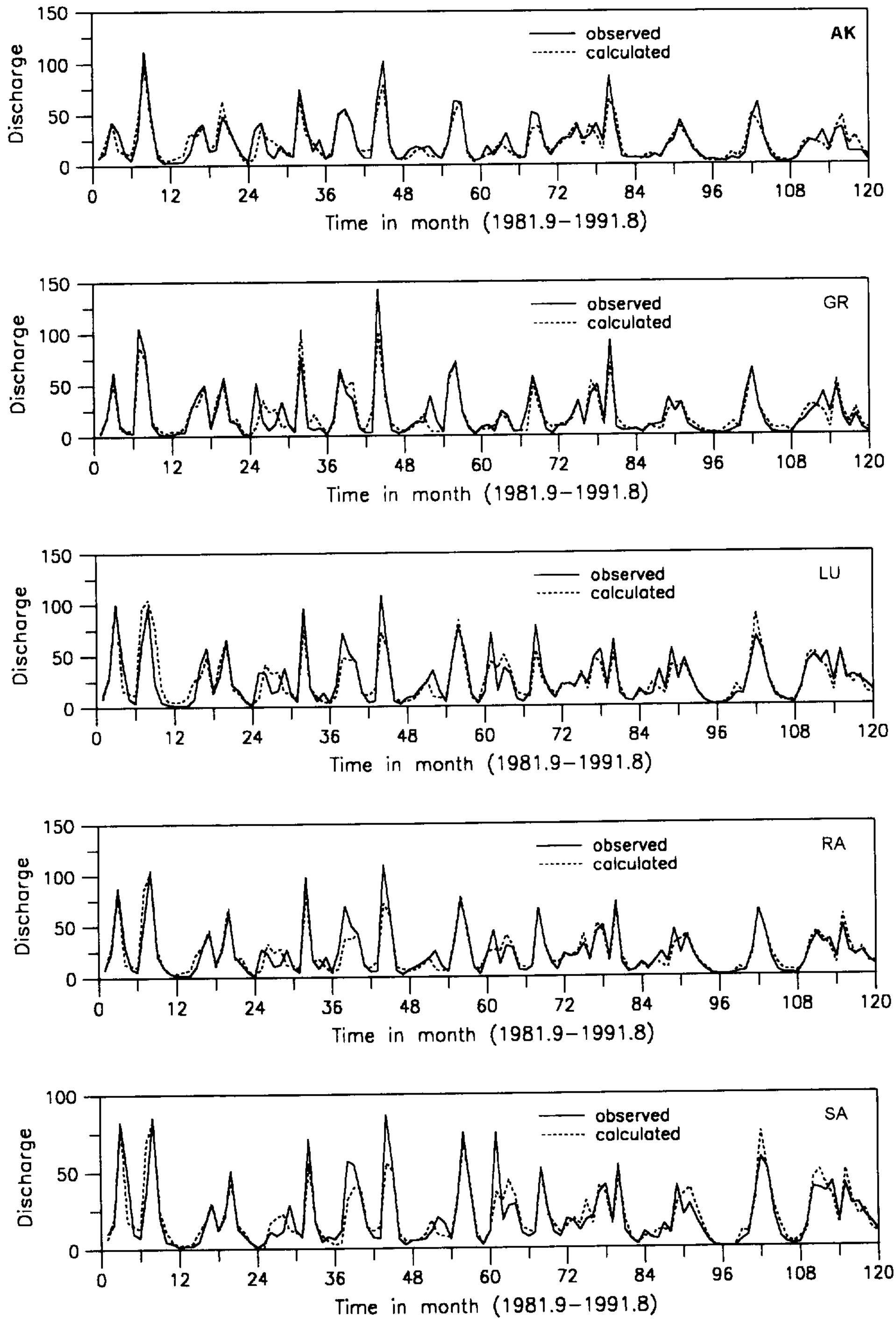
Parameter	Estimate	HWCI
a_1	1.612	0.640
a_2	−2.244	0.680
a_3	0.077	0.060
a_4	0.010	0.002
a_5	0.059	0.021
a_6	0.0042	0.00084

HWCI, half width of a 95% confidence interval.

Table 7
Correlation matrix of parameters for catchment Ha (model 5)

COR*ij*	a_1	a_2	a_3	a_4	a_5	a_6
a_1	1	−0.418	0.092	−0.034	−0.388	0.172
a_2	−0.418	1	−0.129	0.061	0.475	0.006
a_3	0.092	−0.129	1	−0.000	−0.100	−0.007
a_4	−0.034	0.061	−0.000	1	0.213	0.431
a_5	−0.388	0.475	−0.100	0.213	1	0.130
a_6	0.172	0.006	−0.007	0.431	0.130	1

Fig. 9. Residuals vs. time, autocorrelation of residuals, scattergrams of residuals vs. precipitation, temperature and computed discharge diagrams (Ha, model 5).


Discharge
observed
calculated
AK
Time in month (1981.9–1991.8)
Discharge
observed
calculated
GR
Time in month (1981.9–1991.8)
Discharge
observed
calculated
LU
Time in month (1981.9–1991.8)
Discharge
observed
calculated
RA
Time in month (1981.9–1991.8)
Discharge
observed
calculated
SA
Time in month (1981.9–1991.8)

5.1. Estimation

There are different objective functions in use in the literature, depending on the hypotheses postulated relating to the nature of residual u_t. Although it is common to suppose that

$$u_t = q_t - d_t \tag{15}$$

where q_t is observed monthly river flow and d_t is the computed flow. For statistical analysis, it is convenient to have homoscedastic deviations (i.e. common variance σ^2 for all deviations). If this is not the case, a transformation is usually needed. Previous studies (Xu, 1992; Vandewiele et al., 1992, 1993) show that taking a square root transformation is a good hypothesis, i.e.

$$\sqrt{q_t} = \sqrt{d_t} + u_t \tag{16}$$

with

$$u_t \approx N(0, \sigma^2) \tag{17}$$

i.e. u_t is normally distributed with zero expectation and common variance, the so-called model variance. Moreover deviations are assumed to be independent, i.e. for all t

$$Eu_t u_{t-1} = 0 \tag{18}$$

where E is the expectation operator.

The independence of the u_t has been discussed by Vandewiele et al. (1992, 1993); the hypothesis compares with other transformations.

To estimate the parameters the maximum likelihood method was used. Because of the hypotheses in Eqs. (16)–(18), maximising the loglikelihood with respect to the continuous (free) parameters is equivalent to minimising the sum of squares

$$SSQ = \sum_t (\sqrt{q_t} - \sqrt{d_t})^2 \tag{19}$$

where the sum is extended over all months for which output q_t as well as input data p_t and c_t are available.

The quality of minimisation was checked by plotting SSQ versus each of the filter parameters to see whether a global minimum was reached. This was done for every model–basin combination. Illustrations of this procedure can be found in Vandewiele et al. (1992, 1993).

Fig. 10. Computed and observed monthly runoff (mm month^{-1}) series for catchments AK, GR, LU, RA, and SA.

The model standard deviation σ is

$$\sigma = \sqrt{\frac{\text{minimum } SSQ}{N - K}} \tag{20}$$

where N is the number of terms in Eq. (19), and K is the number of filter parameters (regression coefficient).

5.2. Residual analysis

The resulting best model for each catchment was tested in numerous ways by applying statistical methodology including residual analysis. The general behaviour of the residuals is judged by graphs of the residuals versus time, the input variables and computed runoff d_t. The residuals versus time graph is used to check the absence of trend and also homoscedasticity. The scattergrams of residuals versus the other variables p_t, c_t and d_t have to be symmetric with respect to the horizontal axis (zero expectation), and the conditional standard deviation has to be constant (homoscedasticity).

Also, some formal tests are performed. It is checked whether

$$\frac{\bar{u}\sqrt{N - K}}{SDu} \leqslant t(N - K, 5\%) \tag{21}$$

where SDu is the standard deviation of the residual u_t series; N is the number of terms and K is the number of parameters; $t(N - K, 5\%)$ is the critical value of the Student-distribution with $N - K$ degree of freedom and 5% of significant level. The same test but restricted to residuals for each season can be used to check for a seasonal component in the residuals. The 'seasons' are: Autumn (September, October, November), Winter (December, January, February), Spring (March, April, May) and Summer (June, July, August). This check on the seasonality of residuals turns out to be a most severe test of the models.

5.3. Model quality

Evidently, standard deviation σ is an inverse measure of the quality of model performance. In order to measure model quality by a dimensionless quantity, the coefficient of variation of model runoff is used. Vandewiele et al. (1992) show that for mean runoff the latter is equal to

$$mcv = \frac{\sigma\sqrt{4\bar{d} + 2\sigma^2}}{\bar{d} + \sigma^2} \tag{22}$$

where $\bar{d}$ denotes overall mean computed runoff. For small σ (the practical case) this quantity is approximately proportional to σ.

When a basin has a high coefficient of variation of observed runoff (ocv), it can be expected that mcv will be high also. Therefore, a quality measure comparable for different basins is

$$R = \frac{ocv}{mcv} \tag{23}$$

where ocv is defined as $ocv = s/\bar{q}$ with

$$\bar{q} = \frac{1}{N}\sum_t q_t \quad s = \sqrt{\frac{1}{N}\sum_t (q_t - \bar{q})^2}$$

Since mcv is the denominator in Eq. (23), the quantity R is proportional to model quality.

6. Results and discussion

6.1. The best model and good models

The models and the statistical methodology described above were applied to the 11 catchments in the NOPEX region. The best model for each catchment need not necessarily be the same for the other catchments. To find a common model for the region so that the regionalisation of model types and parameter values is possible, good models for each catchments are also determined.

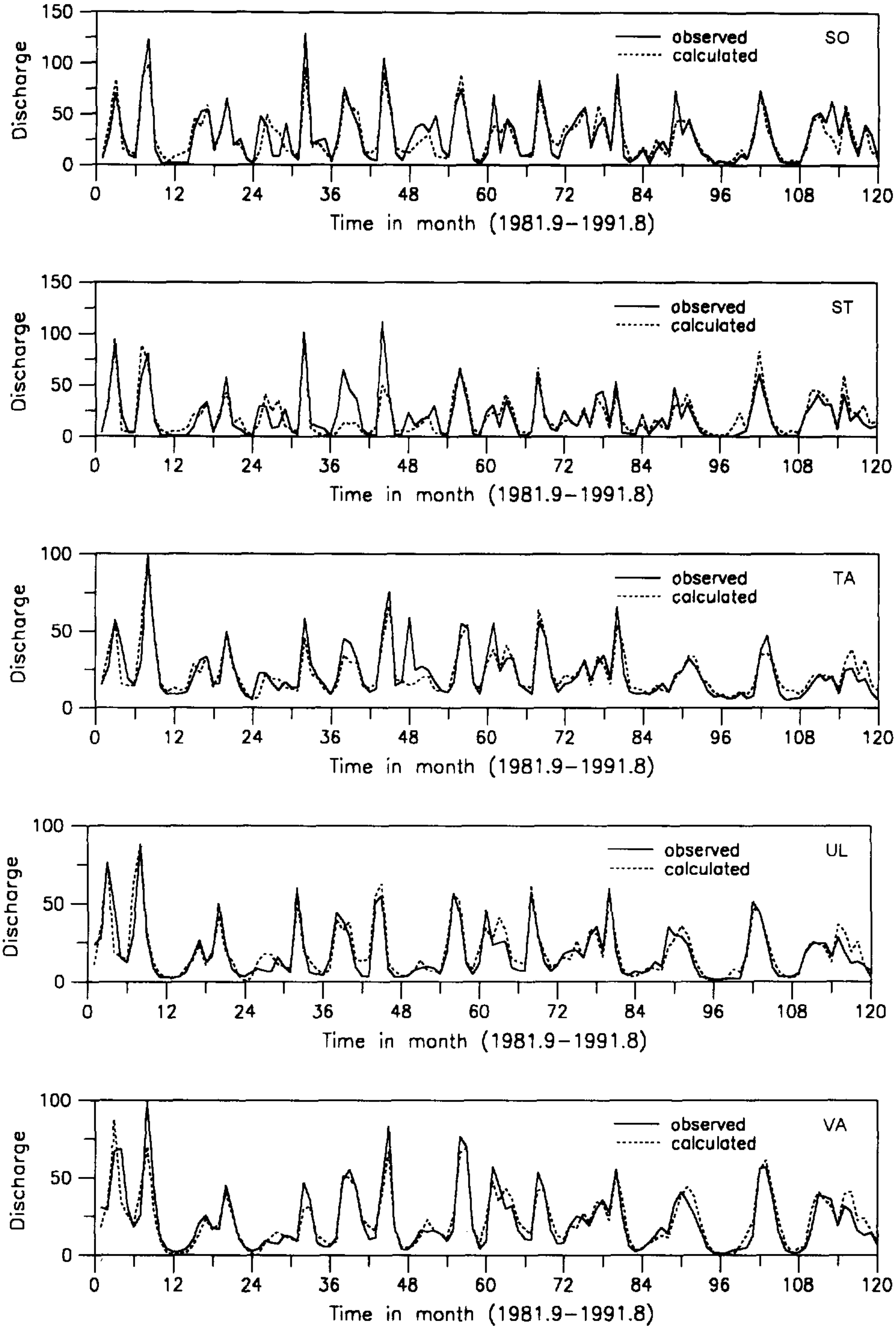
The best model for a given catchment is defined such that the model has maximum quality among those with minimum residual seasonality (number of seasons — out of four seasons — with significant residuals).

The 'good' models were defined to satisfy two conditions, i.e. minimum residual seasonality and quality greater than 90% of maximal quality. Usually there is more than one good model for each catchment.

In the present paper, the eight models compared are obtained by combining evapotranspiration equations and values of discrete parameters b_1 and b_2 as follows:

Model no.	Evap. eqn.	b_1	b_2
1	(8)	1	1
2	(8)	1	2
3	(8)	2	1
4	(8)	2	2
5	(9)	1	1
6	(9)	1	1
7	(9)	2	1
8	(9)	2	2

The eight models were optimised on the 11 catchments (Table 3). Two critical aspects are retained in this table: the quality R as defined in Eq. (23) and the number of seasons (out of four seasons) with significant residuals. It is seen that model quality



Discharge
observed
calculated
SO
Time in month (1981.9-1991.8)
ST
TA
UL
VA

does not differ much among different models for the same basin; seasonality is very well explained. Model 5 can be used for the regionalisation study, since it is one of the good models for almost all the catchments. There is only one season (out of four seasons per year × 11 basin–model combinations equals 44 tests) that shows significant residuals for this model. That means that only $1/44 = 2.3\%$ of the tests are negative. This has to be compared with the 5% significance level, at which the tests were performed. The optimised parameter values of this model are shown in Table 4 for the 11 catchments.

6.2. *A detailed example*

As an example of model output, the results from the river Örsundaån at station Härnevi (305 km^2) are presented here in detail. The catchment is located in the centre of the NOPEX area. The calibration results are summarised in Table 5. Parameter values and their correlations are given in Tables 6 and 7. Graphical outputs are shown in Figs. 8 and 9. It is seen in Tables 6 and 7 that the model parameters are all statistically significant and that their correlations are small in absolute value. Fig. 8 not only shows a very good agreement between computed and observed runoff series, but also gives reasonable estimates of snowfall, actual evapotranspiration, base flow, soil moisture and snow-pack storages. Residuals were random with neither trend nor outlier; residual autocorrelation is insignificant. Furthermore, residuals are homoscedastic (Fig. 9).

Comparisons between the computed and observed monthly runoff series for the other ten catchments are shown in Figs. 10 and 11.

6.3. *Internal aspects of the models*

6.3.1. *Snowfall*

The calculation of how much of the total precipitation falls as snow or rain is of considerable importance for the model performance. While rain is added to soil moisture immediately (and contributes consequently to runoff), snow may remain on the ground surface for some considerable time, say, weeks or even months, before its temperature rises sufficiently to melt into water. Table 8 shows two seasonal aspects: percentage of precipitation which is snow and seasonal snowfall in percentage of annual values. In winter (December, January, February) 69% of precipitation is snow. This value is slightly lower than Eriksson's (1990) result of 72.5%, calculated for the period of 1951–1980. However, the mean temperature over the last 10 years was higher than during the last 30 years. For example, at Uppsala Fly. station the mean winter temperatures are −3.64°C and −4.03°C for

Fig. 11. Computed and observed monthly runoff (mm $month^{-1}$) series for catchments SO, ST, TA, UL and VA.

Table 8
Seasonal snowfall in the NOPEX region (September 1981–August 1991)

Value catchment	Percentage of precipitation that is snowfall					Seasonal snowfall in percentage of annual			
	Autumn	Winter	Spring	Summer	Annual	Autumn	Winter	Spring	Summer
AK	9.2	72.8	15.1	0.0	19	11.7	74.8	13.5	0.0
GR	4.2	64.5	10.5	0.0	14	6.4	83.6	10.0	0.0
HA	5.1	66.0	11.1	0.0	17	7.5	81.8	10.7	0.0
LU	4.1	63.2	11.2	0.0	15	6.3	82.0	11.7	0.0
RA	5.2	66.4	12.3	0.0	17	7.3	80.8	11.9	0.0
SA	4.9	64.4	11.7	0.0	15	8.2	81.6	10.2	0.0
SO	5.2	67.2	11.5	0.0	16	8.1	80.8	11.2	0.0
ST	4.2	67.5	10.7	0.0	14	7.3	83.0	9.7	0.0
TA	8.8	72.1	14.3	0.0	19	10.7	77.2	12.2	0.0
UL	6.6	67.9	13.2	0.0	18	10.3	78.3	11.4	0.0
VA	7.8	70.7	14.4	0.0	21	12.4	80.8	11.3	0.0
Mean	5.9	69.0	12.4	0.0	17	8.7	80.0	11.3	0.0

the periods 1981–1991 and 1961–1990, respectively. Table 8 also shows that the winter period collects 80% of annual snowfall.

6.3.2. Snowmelt

The study of snow melting as a function of time can give important information on the sources of runoff. The percentage of snow melting in each season and the

Table 9
Seasonal snowmelt water and its contribution to streamflow (September 1981–August 1991)

Catchment	Seasonal snowmelt in percentage of annual melt				Percentage of flow that is in the snowmelt				
	Autumn	Winter	Spring	Summer	Autum	Winter	Spring	Summer	Annual
AK	0.0	7.2	92.7	0.0	0.0	18.4	73.6	0.0	29
GR	0.0	11.7	88.3	0.0	0.0	17.1	65.5	0.0	23
HA	0.0	10.8	89.2	0.0	0.0	18.4	68.4	0.0	26
LU	0.0	13.5	86.5	0.0	0.0	21.4	62.2	0.0	24
RA	0.0	12.0	88.0	0.0	0.0	21.1	69.1	0.0	26
SA	0.0	13.2	86.8	0.0	0.0	22.1	69.4	0.0	24
SO	0.0	10.4	89.6	0.0	0.0	18.2	65.7	0.0	24
ST	0.0	13.6	86.4	0.0	0.0	19.8	63.8	0.0	23
TA	0.0	7.9	92.1	0.0	0.0	19.9	71.0	0.0	29
UL	0.0	11.6	88.4	0.0	0.0	24.9	73.6	0.0	29
VA	0.0	11.4	88.6	0.0	0.0	28.9	77.9	0.0	32
Mean	0.0	11.2	88.8	0.0	0.0	20.9	69.1	0.0	26

Table 10
The ratio of actual to potential evapotranspiration for the NOPEX area (September 1981–August 1991)

Autumn	Winter	Spring	Summer	Annual
91%	100%	99%	80%	93%

percentage of total runoff which comes from snow melt are calculated by

$$\% \text{ of flow that is in the snowmelt} = \frac{\text{snowmelt water in the season}}{(\text{snowmelt} + \text{`active' rainfall}) \text{ in the season}} \times 100$$

and are presented in Table 9. It is seen that: (1) 88.8% of snow is melted in the spring season, i.e. from March to May; (2) snow melting water contributes to about 70% of the flow in the spring season.

6.3.3. Evapotranspiration

The ratio of actual to potential evapotranspiration was calculated. For the NOPEX area, the annual value is 93% with a maximum of 100% in winter and a minimum of 80% in summer (Table 10).

6.4. Water balance

The water balance components of the 11 individual catchments and the mean value

Table 11
The computed mean annual water balance of 11 catchments in the NOPEX region (September 1981–August 1991)

Catchment code	Area	Mean prec.		Mean evap.		Mean runoff		Base flow		Fast flow	
	(km^2)	(mm)	(%)	(mm)	(%)	(mm)	(%)	(mm)	(%)	(mm)	(%)
AK	730	733	100	474	65	263	36	102	14	160	22
GR	168	726	100	481	66	246	34	51	7	195	27
HA	305	738	100	452	62	287	39	77	10	209	28
LU	124	750	100	447	60	307	41	98	13	208	28
RA	198	734	100	464	63	272	37	82	11	190	26
SA	727	732	100	494	68	239	33	89	12	150	21
SO	612	729	100	400	55	331	45	92	13	239	33
ST	6.6	693	100	457	66	237	34	31	4	206	30
TA	14	733	100	471	65	264	36	129	18	135	18
UL	950	755	100	520	69	243	32	96	13	147	20
VA	284	750	100	486	65	273	36	146	19	127	17
Mean		734	100	468	64	268	36	90	12	178	24

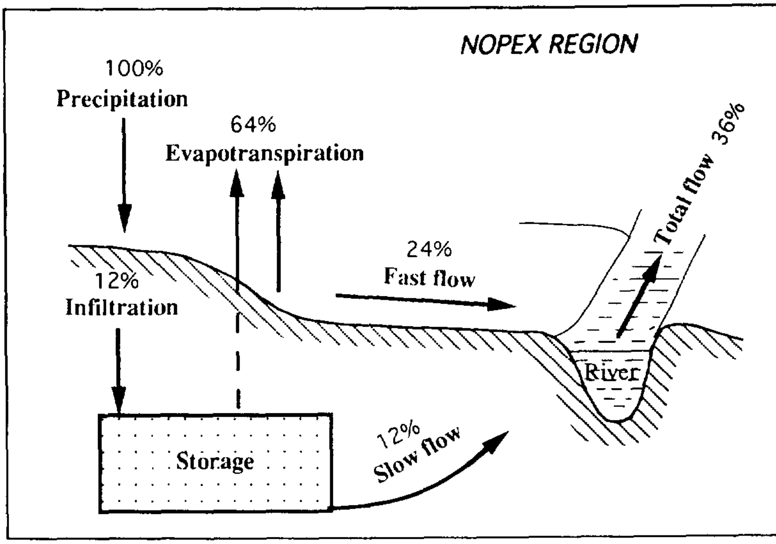


Fig. 12. Graphic representation of water balance of the NOPEX region in percentage of precipitation (September 1981–August 1991).

for the NOPEX region are shown numerically in Table 11 and graphically in Fig. 12. For the NOPEX area, about two-thirds of annual precipitation goes to evapotranspiration, while one-third of precipitation contributes to streamflow. This is consistent with the study of Seibert (1994) of the water balance of the NOPEX area using the long-term average water balance equation. She concluded that: "the water balance in the NOPEX research area was found to include 730 mm precipitation per year which were divided into 470 mm evapotranspiration and 260 mm runoff".

7. Conclusions

To improve knowledge of the hydrological behaviour of small to medium size catchments located in the NOPEX area, a systematic study was performed by using conceptual water balance models. In developing the general model structure for the region, the first conclusion is that for the purpose of water balance studies, simple models using readily available meteorological data as inputs can do the job quite well.

The models were tested on a number of aspects and the following conclusions hold: parameter values are significantly different from zero at the 5% level, correlations between parameters are sufficiently low, residual autocorrelation is non-significant at the 5% level, residuals are homoscedastic, seasonality is very well explained.

The results shown in this paper provide not only valuable reference for the people who are working on the NOPEX project, but also encourage the application of the methodology to other catchments in the Nordic region.

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